Majorana fermions manifested as interface states in semiconductor hybrid structures

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Motivated by recent proposals for the generation of Majorana fermions in semiconducting hybrid structures, we examine possible experimental fingerprints of such excitations. Whereas previous works mainly have focused on zero-energy states in vortex cores in this context, we demonstrate analytically an alternative route to detection of Majorana excitations in semiconducting hybrid structures: interface-bound states that may be probed directly via conductance spectroscopy or scanning tunnel microscope measurements. We estimate the necessary experimental parameters required for observation of our predictions.

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I. INTRODUCTION

The prediction[1,2] and experimental observation[3,4] of topological insulators has triggered an avalanche of research activity. Besides a number of fundamentally interesting aspects of the quantum spin Hall effect[5] appearing in such systems, this class of materials also harbors a very real potential in terms of practical use in quantum computation. The reason for this is that they have been shown to host so-called Majorana fermions[6] under a variety of circumstances.[7–11] Such excitations satisfy non-Abelian statistics which form a centerpiece in recent proposals for topological quantum computations.[12]

From a technological point of view, the field of topological insulators is still in its infancy. Two recent works[13,14] that addressed the generation of Majorana fermions in *semiconducting* devices have therefore attracted much attention since semiconductor technology is very well developed and thus offers greater experimental control over the system. The experimental setups suggested by Sau *et al.*[13] and Alicea[14] are shown in Figs. 1(a) and 1(b), respectively. Common for both proposals is that a quantum well with Rashba and/or Dresselhaus spin-orbit coupling is contacted to a superconducting reservoir and then driven into a topological phase by means of a magnetic field. When the latter exceeds a critical threshold, it effectively renders the band structure in the quantum well formally equivalent to a spinless k_x+ik_y superconductor. This is a system which is known to host zero-energy Majorana fermions in vortex cores.

Up to now, it is precisely the prospect of Majorana fermions residing in vortex cores that has constituted the bulk of proposals for a realization of this exotic class of excitations in a condensed-matter system (see, however, e.g., Refs. 11, 15, and 16). However, as we will show in this work, the Majorana fermions may also leave a distinct signature in semiconducting hybrid structures as the ones shown in Fig. 1. Namely, interface-bound states with a unique dispersion which may be probed directly via conductance spectroscopy or scanning tunnel microscope (STM) measurements. To demonstrate this, we will first proceed to establish a direct correspondence between the systems considered in Fig. 1 and a spinless k_x+ik_y superconductor, and then calculate the energy dispersion for the interface-bound states analytically. The fingerprint of these states in STM measurements would constitute a clear-cut experimental observation of Majorana excitations in a condensed-matter system.

Both the presence of spin-orbit coupling and a Zeeman interaction are key ingredients in establishing a topological superconducting phase in the systems suggested by Sau *et al.*[13] and Alicea.[14] The spin-orbit coupling ensures that a singlet-triplet mixing occurs for the induced superconducting order parameter, and thus generates a spinless p-wave order parameter. Upon introducing a Zeeman field, one of the pseudospin bands is raised above the Fermi level and one is left with a single-band spinless p-wave superconductor. Whereas such a Zeeman field would have to be enormous in a conventional metal, the high-g factor and tunable Fermi level in semiconducting devices makes this possible even at fields below 1 T. An additional advantage of this is that the applied field then also remains well below the critical field H_c for the proximity superconductor, which in many materials far exceeds 1 T.[17]

The purpose of this work is to demonstrate a clear experimental signature of the Majorana excitations proposed to exist in the setups of Refs. 13 and 14, which also provides an alternative route to observation of Majorana fermions compared to the standard proposal of zero-energy vortex states. Our result applies both to Figs. 1(a) and 1(b) but for the sake of not overburdening, this work with analytical calculations we here focus on the setup in (a) which yields the most transparent results. The system in Fig. 1(a) consists of (i) an s-wave superconductor, preferably with a high T_c such as Nb, (ii) a quantum-well semiconductor with Rashba spin-orbit coupling, such as InAs, and (iii) a ferromagnetic insulator such as EuO.

II. THEORY

The Hamiltonian for the conduction band of the quantum well then reads

$$\mathcal{H}_{QW}=[-\nabla^2/(2m')-\mu]\hat{1}-i\alpha(\partial_y\hat{\sigma}_x-\partial_x\hat{\sigma}_y), \qquad (1)$$

where $\hat{\ldots}$ denotes a 2×2 matrix in spin space. Here, m' is the effective mass of the electron (typically $m'\simeq m_e/20$) whereas α denotes the spin-orbit coupling constant. By means of the proximity effect to a ferromagnetic insulator, a Zeeman field couples to the spins via

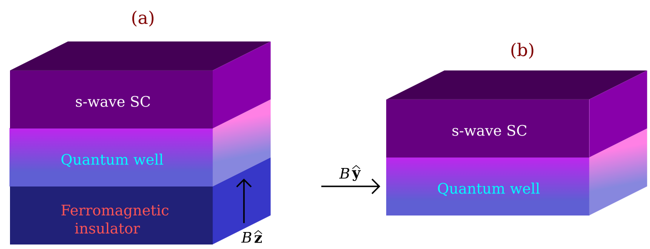

FIG. 1. (Color online) The experimental setup proposed in Refs. 13 and 14 for generation of Majorana fermions in a semiconducting hybrid structure. In (a), a superconducting order parameter and Zeeman interaction is induced by means of the proximity effect in a quantum well with Rashba spin-orbit coupling whereas in (b), the quantum well features a combination of Rashba and Dresselhaus spin-orbit coupling with an exchange interaction induced by an external field rather than a ferromagnetic insulator.

$$\mathcal{H}_{FI} = -V_z\hat{\sigma}_z, \tag{2}$$

where V_z is the magnitude of the exchange splitting. This interaction is strongly reduced compared to its value in the bulk ferromagnetic insulator, and it is thus reasonable to expect a magnitude of order $\mathcal{O}$ (millielectron volt). The band structure in the quantum well may now be obtained by diagonalizing the total Hamiltonian,

$$\mathcal{H} = \mathcal{H}_{QW} + \mathcal{H}_{FI}, \tag{3}$$

which yields two pseudospin bands,

$$\mathcal{E}_{\boldsymbol{k}}^{\beta} = k^2/(2m') - \mu + \beta\sqrt{\alpha^2k^2 + V_z^2}, \quad \beta = \pm 1. \tag{4}$$

Before introducing the superconducting proximity effect, it is instructive to pause briefly to consider the band structure Eq. (4) in more detail. It follows that when the exchange interaction exceeds the chemical potential, $V_z > \mu$, the upper band is raised above the Fermi level for all momenta, i.e., $\mathcal{E}_{\boldsymbol{k}}^{+} > 0$. This is illustrated in Fig. 2, where the dashed line indicates the Fermi level.

On the other hand, the lower band crosses the Fermi level at the momentum,

$$k_F = [2m'(m'\alpha^2 + \mu + \sqrt{m'\alpha^2(m'\alpha^2 + 2\mu) + V_z^2})]^{1/2}. \tag{5}$$

Enter now the superconducting pair field generated by the proximity s-wave superconductor. It adds a term to the Hamiltonian expressed by the original spinors $\psi = [\psi_\uparrow, \psi_\downarrow]$,

$$\mathcal{H}_{\text{SC}} = \int d^2\boldsymbol{r}[\Delta\psi_\uparrow^\dagger(\boldsymbol{r})\psi_\downarrow^\dagger(\boldsymbol{r}) + \text{H.c.}]. \tag{6}$$

Transforming the above equation into the new pseudospin basis of the long-lived excitations at Fermi level then produces the following gap for the lower band:[14]

$$\Delta_{\boldsymbol{k}} = -\alpha\Delta(k_y - ik_x)/(2\sqrt{V_z^2 + \alpha^2k^2}). \tag{7}$$

The Hamiltonian can now be written as

$$\mathcal{H} = \int d^2\boldsymbol{k}\,\phi_{\boldsymbol{k}}^\dagger\mathcal{M}_{\boldsymbol{k}}\phi_{\boldsymbol{k}}, \tag{8}$$

where $\phi_{\boldsymbol{k}} = [\varphi_{\boldsymbol{k}}, \varphi_{-\boldsymbol{k}}]$ is the pseudospin basis while

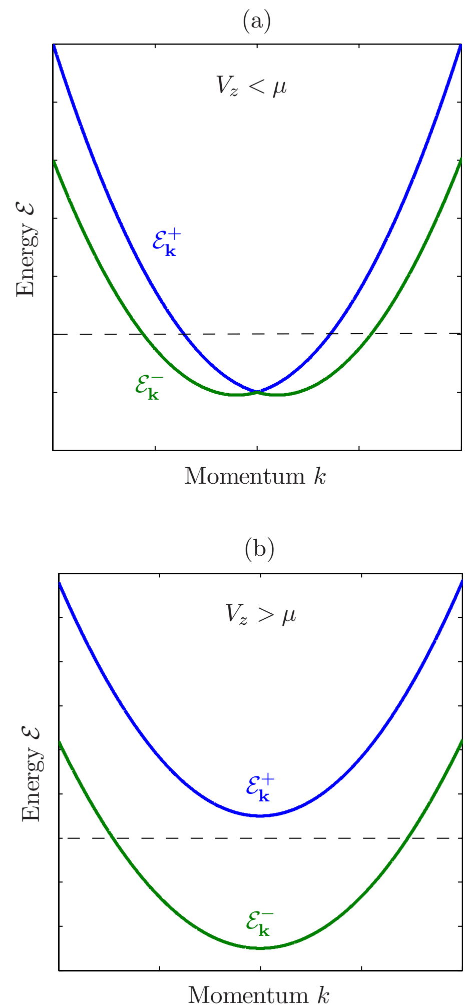

FIG. 2. (Color online) Qualitative picture of the pseudospin bands in Eq. (4). (a) When the chemical potential is larger than the induced Zeeman splitting, both bands cross the Fermi level. (b) When the Zeeman splitting exceeds the chemical potential, only the lower band crosses the Fermi level.

$$\mathcal{M}_{\boldsymbol{k}} = \begin{pmatrix} \mathcal{E}_{\boldsymbol{k}} & \Delta_{\boldsymbol{k}} \\ \Delta_{\boldsymbol{k}}^* & -\mathcal{E}_{\boldsymbol{k}} \end{pmatrix}. \tag{9}$$

Here, we have defined $\mathcal{E}_{\boldsymbol{k}} \equiv \mathcal{E}_{\boldsymbol{k}}^{-}$ and the $\hat{\sigma}_j$ matrices now operate in pseudospin space.

At this point, we can formally identify the obtained Hamiltonian as fully equivalent to a spinless $k_x + ik_y$ superconductor (after a gauge transformation of $e^{i\pi/2}$). We now proceed to demonstrate that the Majorana states in this system leave a unique fingerprint not only as zero-energy states in a vortex core but also as *interface-bound states*. Presumably, this simplifies greatly their experimental detection since one avoids the need to generate vortices in the quantum well. Instead, it suffices to probe the surface density of states (DOS) at the edge of the quantum well either via conductance spectroscopy or STM measurements.

III. RESULTS

To be definite, let us consider the edge defined by $x=0$ (although our results are qualitatively identical for the edge

$y=0$). Starting from the Hamiltonian Eq. (8), we construct the wave function in the quantum well which at $x=0$ takes the form

$$\Psi(x=0) = c_1\begin{pmatrix} u_{\boldsymbol{k}} \\ v_{\boldsymbol{k}} e^{-i\gamma_{\boldsymbol{k}}^+} \end{pmatrix} + c_2\begin{pmatrix} v_{\boldsymbol{k}} e^{i\gamma_{\boldsymbol{k}}^-} \\ u_{\boldsymbol{k}} \end{pmatrix}, \tag{10}$$

where we have defined

$$e^{i\gamma_{\boldsymbol{k}}^{\pm}} = -(k_y \mp ik_x)/k_F \tag{11}$$

and introduced the ratio between the coherence factors,

$$\frac{u_{\boldsymbol{k}}}{v_{\boldsymbol{k}}} = e^{ia\cos(\varepsilon/|\Delta_{\boldsymbol{k}}|)}. \tag{12}$$

The constants $\{c_1, c_2\}$ are unknown and must be determined by proper boundary conditions. At the vacuum edge $x=0$, the wave function must vanish and we thus demand

$$\Psi(x=0) = 0, \tag{13}$$

which allows for a determination of $\{c_1, c_2\}$. Doing so, we find that a nontrivial solution is obtained if the criterion

$$\begin{vmatrix} e^{i\beta} & e^{i\gamma_{\boldsymbol{k}}^-} \\ e^{-i\gamma_{\boldsymbol{k}}^+} & e^{i\beta} \end{vmatrix} = 0 \tag{14}$$

may be satisfied. This is indeed the case when

$$|\varepsilon/\Delta| = \frac{\alpha k_F|\sin\,\theta|}{2\sqrt{V_z^2 + \alpha^2 k_F^2}}, \tag{15}$$

where k_F was defined previously. This equation describes precisely the announced interface-bound states and is one of the main results in this work. In general, subgap resonant states are manifested as an enhanced DOS/peak structure in such measurements whereas the rest of the subgap DOS would be suppressed due to the fully gapped Fermi surface. An important point to note is that since the present interface state in Eq. (15) is strongly dependent on the angle of incidence relative the edge, one would expect that the DOS to be enhanced in large parts of the subgap regime rather than featuring sharp spikes at isolated energies. Qualitatively, this would be experimentally seen as a broad humplike enhancement of the low-energy conductance or surface DOS, similarly to the proposed chiral p-wave state in Sr_2RuO_4.[18] We note that a quasiclassical treatment of the above wave functions and interface-bound state would require the proximity-induced superconducting gap to be much smaller than the Fermi level. However, the present treatment does not rely on such an assumption.

We now analyze the behavior of this interface state using a realistic set of experimental parameters to identify the relevant energy regime where it resides and thus may be probed by, e.g., STM measurements. The general requirement for the mapping to the spinless k_x+ik_y-wave state is that V_z exceeds μ in magnitude. In addition, it would be desirable to maximize the Fermi momentum k_F to obtain a large normal-state DOS for the benefit of superconducting pairing. Considering Eq. (5), it is seen that this can be obtained either via a large V_z or large $m'\alpha^2$. The magnitude of V_z will be largely

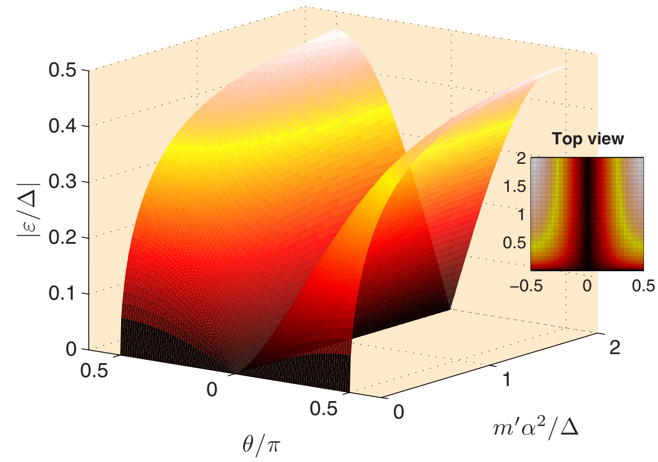

FIG. 3. (Color online) Dispersion of the interface-bound state as a function of the angle of incidence (θ) and the normalized spin-orbit coupling strength ($m'\alpha^2/\Delta$). Here, we have fixed $V_z/\Delta=2$, $\mu/\Delta=3/2$, as should be experimentally viable for a proximity-induced gap of size $\Delta=0.5$ meV. The experimental signature of this interface state would be an enhanced subgap DOS, in particular, near the Fermi level, compared to the otherwise fully suppressed DOS within the gap in the absence of such states.

determined by the interface properties (such as lattice mismatch) of the ferromagnetic insulator but values up to a few millielectron volt should be within experimental reach.[19] The spin-orbit coupling strength can to some extent be controlled by a gate voltage, as demonstrated in, e.g., Ref. 20, bordering toward 1 K in InGaAs quantum wells. As mentioned previously, the proximity-induced superconducting gap will also be substantially reduced compared to its bulk value in the s-wave superconductor, and a reasonable estimate would be $\Delta \simeq 0.5$ meV. As a very moderate estimate, we then fix $V_z = 1$ meV and set $\mu=0.75$ meV; the latter is tunable in a controlled fashion. With these parameters, we now plot the interface state versus the angle of incidence θ and the normalized spin-orbit coupling strength $m'\alpha^2/\Delta$ in Fig. 3. As seen, the energy increases with $m'\alpha^2/\Delta$ and eventually saturates around 0.5Δ. In this plot, we have considered values of $m'\alpha^2/\Delta$ up to 2 in order to demonstrate the evolution of the interface state in the limit of large spin-orbit coupling. Such values may be accessed in a scenario where the proximity-induced gap is very small, e.g., $\Delta \leq 0.05$ meV. For the present choice of parameters, the maximum value of $m'\alpha^2/\Delta$ attainable lies around 0.10–0.15. As seen from the plot, the energy of the interface state is small in this regime, $|\varepsilon/\Delta| \ll 1$, and reaches zero at normal incidence. This should be readily observable in local DOS measurements at the surface of the quantum well, which routinely probe structures with energy resolution down to $\simeq 200$ μV.[21]

So far, we have established the presence of interface-bound states in semiconducting hybrid structures as shown in Fig. 1 by utilizing an exact mapping onto a spinless k_x+ik_y superconductor model in a realistic parameter regime. The experimental signature of these interface state would be an enhanced low-energy (below the gap) DOS compared to the otherwise fully suppressed DOS within the gap in the absence of such states. However, there are certainly experimental challenges associated with the proposed structures which we would like to acknowledge here. One point, which in particular pertains to the setup in Fig. 1(a), is related to the Meissner response of the superconductor due to the ferromagnetic insulator. This can be avoided by utilizing a ring-like structure (as in Ref. 13) of the superconducting host material which would suppress the orbital effect. In this sense, the structure in Fig. 1(b) is beneficial since the field here resides in the plane of the quantum well, thus strongly suppressing the orbital response. As previously mentioned, another challenge is to achieve a sufficiently good interface coupling between the quantum well and the ferromagnetic insulator in order to have an appreciable magnitude of the Zeeman field V_z. In this context, we note that EuO has previously been contacted to Al with a successfully induced Zeeman field in Ref. 19 as probed by conductance spectroscopy, which demonstrates that such a procedure should be feasible.

We conclude by mentioning some possible future venues of investigation that might prove useful. One important aspect related to the appearance of Majorana surface states is the role of finite-size effects in the system. Such effects can cause an overlap between the surface wave functions on opposing edges of the sample and thus leading to a strongly modified excitation spectrum near the interface. This has very recently been investigated in the context of the topological insulators HgTe/CdTe and also in Bi_2Se_3.[22–25] Extending such considerations to the present hybrid semiconductor/superconductor structure would be of interest. Another point that might be worth investigating is how the formation of Majorana fermions would be altered when replacing the conventional s-wave superconductor considered in the present work with a more exotic material, such as, e.g., a d-wave high-T_c cuprate superconductor or a p-wave triplet superconductor such as Sr_2RuO_4. In such a scenario, one might expect a nontrivial interplay between Andreev bound states pertaining specifically to the order parameter of the superconductor and the induced Majorana interface states in the two-dimensional electron gas (2DEG) region. Such considerations were very recently due in the context of topological insulators,[11] and might also be useful in the present scenario. Finally, we note that the interplay between topological order and superconductivity, spin-orbit coupling, and magnetism has recently been studied by several authors[26,27]

IV. CONCLUSION

In summary, we have investigated an alternative route for experimental observation of Majorana states in semiconducting hybrid structures compared to the previously proposed vortex-core states. This route consists of probing interface-bound states via conductance spectroscopy or STM measurements, which we have analytically demonstrated the existence of in this work. With a conservative estimate for experimental parameters, we find that these interface states reside on an energy scale which should be clearly resolvable in such measurements. Whereas there are still considerable technological challenges regarding the detection of Majorana fermions in topological insulators, pertaining, e.g., to producing materials of sufficiently high quality, the virtue of the present proposal is that semiconductor technology is very well developed and thus could lead to the experimental observation of Majorana fermions as interface-bound state when utilizing present-day methods.

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